Radio Interferometry Fundamentals

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So far discussed only single dish radio/mm obs

- Resolution $\lambda/D$, for $D=20m$, is 30’’ at mm-wavelengths and 30’ (diameter of full moon) at cm (i.e. $\lambda = 18cm$)

- Paradox - In fact despite radio/mm having the longest wavelength it can achieve the highest angular resolution of any wavelength (up to 1 mas = 0.001’’).

- Reason - Interferometry is relatively easy to do at radio and mm wavelengths

- Presently at cm 5% science single dish and 95% interferometry, at mm more 50%/50% but with ALMA interferometry will dominate at mm as well.
Very Large Array (VLA)
VLA Image – Radio Galaxy Cygnus A
(500 million light years away)

Radio astronomers can make nice images too
ALMA Atacama Large Millimetre Array

at 5000m elevation, Atacama Desert, Chile

50 times 12m diameter dishes, moved around on 200 pads

Myself at the site in April 2001
ALMA Configuration layout – John Conway
Very Long Baseline Interferometry (VLBI)

Make an interferometer the size of the Earth – combine signals from antennas all around Earth – Tape record data and combine later ‘off-line’ or send in real-time over internet
Square Kilometer Array (SKA)

Future cm wavelength interferometer array

International project, estimated cost 1.5 B€ phase 2 ready by 2020
Inner SKA core

Parabolas + aperture arrays, ...
SN1993J in M81
VLBI Observations

J.M. Marcaide, A. Alberdi, E. Ros,
et al.

© J.M. Marcaide, Universitat de València, 2000
Japanese 6m antenna on VSOP satellite orbits the Earth out to 4 Diameters increasing the resolution of VLBI
Michelson Stellar Interferometer

First direct measurement of a stellar diameter.
Michelson and Pease (1921)
Very Large Telescope Interferometer (VLTI)  
- Paranal, Chile – optical interferometer

Aerial View of Paranal Observing Platform with VLTI Light Paths

ESO PR Photo 108/01 (18 March 2001) © European Southern Observatory
Radio Multiplying interferometer

Quarter period delay

Interferometer response to a point source brightness \( I_o \) an angle \( \theta \) from zenith

Form complex visibility \( V = V_R - iV_I \). Amplitude of that visibility depends only on point source strength and phase on position

\[ V_R = I_o \cos(2\pi u \theta) \]

\[ V_I = I_o \sin(2\pi u \theta) \]

\( u = b/\lambda \) baseline length in wavelengths
2D Response function of Interferometer Baseline close to the zenith is

Sky Response Function at zenith

cos(2π(ux))

Real channel response function at zenith. 'Corrugated Roof' with spatial frequency u,v
If the two telescope are not East-West

Real Resonse Function on sky

\[ \cos(2\pi(ux + vy)) \]

Telecope layout

\[ u_r = \sqrt{u^2 + v^2} \]

Real channel response function at zenith. 'Corrugated Roof' with spatial frequency \( u, v \)
Interferometer outputs 2 numbers, signal in real channel \((V_r)\) and in imaginary \((V_i)\), together give complex visibility \(V(u,v)\) – this equals the 2D FT of \(I(x,y)\) evaluated at a \(u,v\) given by the projected baseline coordinates EW and NS measured in wavelengths.

Have many baselines can collect lots of information about the 2D FT of the source.
To observe not at zenith and track source across sky add delay $T$ to compensate extra geometrical path, equivalent to moving an antenna so its in the plane perpendicular to ray.

Electronic delay moves centre of fringe pattern so its always centred on the source, and hence ‘tracks’ its position. Fringe spacing on the sky depends on PROJECTED BASELINE measured in wavelengths $u = b' / \lambda$ as seen from source. Rotation of Earth helps us collect more data.
Earth Rotation
East-West baselines
Source at declination
Delta = +30

View of East-West baseline as Earth rotates

uv tracks are ellipses – squashed by cos(delta), cannot usually observe for 24hrs because most sources not above horizon that long, but if above horizon for longer than 12hrs can get complete ellipse using hermitian property.
1974 Nobel Prize in Physics to Martin Ryle (Cambridge) for development of Aperture Synthesis

uv coverage and imaging simplest for E-W arrays, but also have some disadvantages
Example – Small VLBI array, gives an irregular uv coverage. Each baseline gives an elliptical track, but not centred on origin.
Interferometry Difficulty 1 - The Inverse Problem

We know how visibility is related to source structure

\[ V(u,v) = \bar{I}(u,v) = \int \int I(x,y) \exp(-2\pi i(ux + vy)) \, dxdy \]

If we had complete information on \( V(u,v) \) at all \( u,v \) then inverting the problem is easy

\[ I'(x,y) = \int \int V(u,v) \exp(2\pi i(ux + vy)) \, dudv \]

Unfortunately we don’t know value of \( V(u,v) \) everywhere.
‘Dirty Imaging’

\[ V'(u,v) = S(u,v)V(u,v) \]

Where \( V(u,v) \) is the true visibility \( S(u,v) \) the sampling function and \( V'(u,v) \) the sampled visibility

The visibility is related to the image via
\[ V(u,v) = FT(I(x,y)) \]

If we take the IFT of \( V'(u,v) \) then using the convolution theorem we get
\[ IFT(V') = IFT(S) \ast FT(V) \]

where \( \ast \) indicates convolution

\[ IFT(V') = B(x,y) \ast I(x,y) \]

where \( B(x,y) = IFT(S(u,v)) \) is the 'dirty beam' and \( \ast \) is convolution (copying of the beam, at every point on the source).
The Dirty Image

Dirty beam = The PSF

FT

True image

Dirty Image = True Image convolved with dirty beam
Need for Deconvolution

Extrapolate inward

Interpolate

Extrapolate Outward

To improve image further must both ’interpolate between measured visibilities and ’extrapolate’ beyond outer edge and into inner hole. ’Interpolation’ removes effects of far sidelobes and the ’Extrapolation’ that of Inner sidelobes and central bowl.
Find peak in image $I_{\text{max}}$

Remove the effect of a delta function of flux $gI_{\text{max}}$ at position of peak

Add a delta function of flux at $gI_{\text{max}}$ at position of peak in 'Clean Components' file

Loop gain $'g'$ where $g$ is in range $0$ to $1$

Interate until nothing but noise is left in 'residual' image - usually convolve 'clean components' by a gaussian 'restoring' beam

CLEAN Algorithm (Nordic invention Högbom 1974 – one of top 10 cited AA papers)
Example – in ID, Gain = 0.5

Residual Image

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<th>Iteration Number</th>
</tr>
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<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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</tbody>
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Clean components
The Dirty Image

Dirty beam

The PSF

True image

Dirty Image

FT

The PSF

UV-coverage

FT

Dirty beam
Clean: Example

Clean components convolved with Gaussian ‘restoring beam’ (X=FWHM), our estimated source structure at resolution X

FT of Clean components, model visibility

FT of restored image
Clouds of water vapour in atmosphere or ionised gas in ionosphere cause the ‘refective index’ (n) for radio waves to be different from 1, hence radio waves travel slightly slower than the speed of light by a factor 1/n.

A ray passing a length $L$ through a cloud causes a change in phase compared to free space propagation of

$$\Delta \phi = \frac{2\pi L(n - 1)}{\lambda}$$

Interferometry Difficulty 2 – Phase errors
Visibility output depends on relative Phase of E-field on wavefront, in this case zero.

If there is cloud above one antenna which delays signal then relative phase of E-field on wavefront and hence visibility phase effected.
Fourier Phase Is more important than Amplitude for finding source structure
Closure Phase

Consider the measured phase of the visibility $\theta'$ and the true phase of the visibility $\theta$ on each baseline

$$\theta'_{12} = \theta_{12} + \Delta \phi_1 - \Delta \phi_2$$

$$\theta'_{23} = \theta_{23} + \Delta \phi_2 - \Delta \phi_3$$

$$\theta'_{31} = \theta_{31} + \Delta \phi_3 - \Delta \phi_1$$

We can form a 'closure phase' by adding the phase on a closed loop of baselines

$$C_{123} = \theta'_{12} + \theta'_{23} + \theta'_{31} = \theta_{12} + \theta_{23} + \theta_{31}$$

This 'Closure phase' depends only on the source structure and is independent of the atmosphere!

(Re)discovery of closure phase idea made maging on baselines >5km possible.

Two approaches – closure phase, combine measurements to make quantity independent of atmosphere, or solve explicitly for antenna-based phase errors and correct – must be done at same time we deconvolve, needs iterative methods
- **Image without self-calibration**

  - Phase calibration using nearby source observed every 20 minutes
  - Peak ~ 22Jy
  - Display shows -0.05Jy to 0.5Jy
After 4 amplitude and phase calibrations
To work need a bright target, if our target weak wwitch back an forth every few minutes between the target source and nearby (few degrees away) bright, compact calibrator. Use self-cal methods to estimate atmospheric phase/amplitude toward calibrator source (which is bright and often simple) – interpolate in time and apply these corrections to data for a target source (which is maybe weak and complicated).
Model linear methanol feature as Keplerian rotating disk around a 30Msol protostar. Massive stars form in the same way as low mass stars by accretion disks.

Pestalozzi, Elitzur, Conway, 2009 arXiv:0904.3722

NGC7538-IRS1N VLA obs

VLBI methanol maser observations ~1mas resolution