Heat Load from the Internal NOTCam Motor Wires

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March 3, 1998

Introduction

An investigation has been carried out to see if the problem of the high heat load from the NOTCam motor wires can be reduced by using an alternative material for the wires. A number of metals have been identified as possible candidates and their thermal conductive and resistive properties evaluated.

The exercise is to try and find a possible material that would reduce the thermal conductivity but not at the cost of high resistivity and hence resistive (I^2R) losses. The two dimensional parameters of the wire namely the cross-sectional area and length can be adjusted to find an optimum combination. The thermal losses are inversely proportional to the length and proportional to the cross-section but the resistive losses are the exact opposite.

In connection with the resistive power losses another property that can be exploited is the fact that the current flows through the cables only for a fraction of the time, i.e. only when the wheels are driven. Also the amount of current can be optimised since most of the motors don't need to be driven at their maximum torque.

Thermal conductivity of the wires

Definition. Thermal conductivity is the time rate of heat transfer by a conductor, through unit thickness, across unit area for unit difference in temperature.

If the two opposite faces of a rectangular solid are maintained at a temperature t_1 and t_2 the heat conducted across the solid of a section A and thickness l in a time T will be,

$$Q = \frac{K(t_1 - t_2)AT}{l} \tag{1}$$

K is a constant depending on the nature of the substance, designated as the specific heat conductivity, K is usually give for Q in calories, t_1 and t_2 in ^{o}C, A in cm^2, T in sec, and l in cm. A Calorie is equal to 860 watt hours or 4.19 joules.

From equation 1 it can be seen that the thermal losses in a wire are proportional to the cross-sectional area and inversely proportional to the length. Therefore the ideal cable should be very long and thin and with a small thermal conductivity.

The typical variation of thermal conductivity against temperature is small for most metals in the temperature range 80k to above 300K though as can be seen in figure 1 it does rise rapidly below 80K. The operating temperatures in the NOTCam dewar are above 80K so the rise in the curves will be ignored.

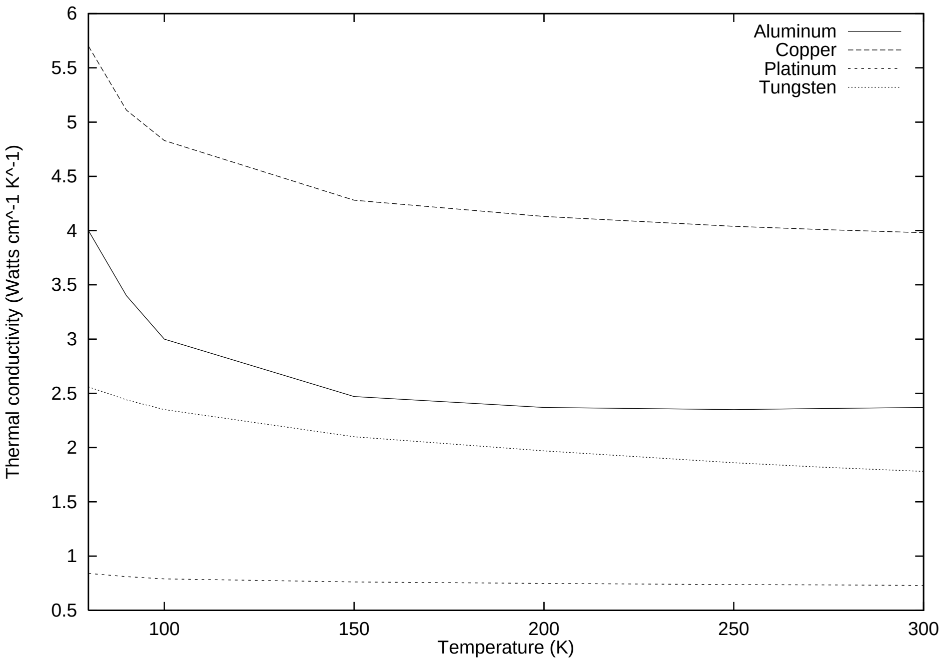

Figure 1: Variation of thermal conductivity for a random selection of metals against temperature in the range 80K to 300K.

Given in table 1 is a list of a large number of pure metals (plus the alloy constantan) with their thermal conductivity at 100K. The table shows that for thermal conductivity copper (the usual wire material) is one of the worse you could choose and constantan the best. Later it will be shown that the resistive properties of constantan are not so desirable.

Table 2 shows some potential metals that could be used for conductors and their variation in thermal conductivity with temperature. The numbers for constantan are approximations since only the values for 80K and 300K are available but this material has an extremely low thermal coefficient.

If we assume that a realistic average length for the motor cable is 100cm and given the temperature gradient of 220K (80K to 300K) and that there are eight motors each with ten wires then using equation 2 a plot of the dependence of cross-sectional area against thermal conduction can be produced for a selection of wire materials.

The thermal load is calculated using;

$$Q = \frac{K_{200K}(t_1 - t_2)AN_wN_m}{l} \tag{2}$$

where K is the thermal conductivity in $\mathrm{Wcm^{-1}K^{-1}}$, A is the cross-sectional area, N_w is the number of wires per motor, N_m is the number of motors, $(t_1 - t_2)$ is the temperature difference, and l is the length of each wire.

From figure 2 it can be seen that constantan is easily the best material to use to minimise the thermal conduction, followed by nickel. Even for quite a large cross-sectional area of $1\mathrm{mm}^2$ constantan only gives a couple of hundred milli-watts compared to copper with say a cross-section of $0.1\mathrm{mm}^2$ gives approaching one watt for all eighty motor wires. These values now need to be compared against the resistive losses of these and other materials.

Metal	Thermal Conductivity, (Watts/cm K)
Aluminium	3.00
Antimony	0.23
Beryllium	4.1*
Bismuth	0.115*
Carbon	†
Cadmium	1.03
Chromium	1.58
Constantan	0.2*
Copper	4.83
Gold	3.45
Iridium	1.51*
Iron	1.32
Lead	0.396
Magnesium	1.69
Mercury	0.33*
Molybdenum	1.79
Nickel	1.58
Niobium	0.552
Palladium	0.69*
Platinum	0.79
Silver	1.50
Tantalum	0.592
Tin	0.85
Titanium	0.312
Tungsten	2.35
Zinc	1.32

Table 1: Thermal conductivity of pure metals at 100K, except for items marked with a * which are at -100 ^{o}C. †Due to the large number of forms Carbon appears in (diamond to graphite with very large differences between // and $\perp$ to the axis of extrusion) no single value can be given.

Metal	Thermal Conductivity, (Watts/cm K)					
	60K	80K	100K	150K	200K	273K
Aluminium	6.7	4.0	3.0	2.47	2.37	2.36
Constantan	0.2	0.2	0.2	0.21	0.21	0.22
Copper	8.5	5.7	4.83	4.28	4.13	4.01
Gold	3.8	3.52	3.45	3.35	3.27	3.18
Iron	2.65	1.68	1.32	1.04	0.94	0.835
Manganin	0.097	0.120	0.133	0.156	0.172	0.206
Platinum	1.01	0.84	0.79	0.762	0.748	0.734
Silver	5.5	4.71	4.50	4.32	4.3	4.28
Tungsten	3.18	2.56	2.35	2.1	1.97	1.82

Table 2: Thermal conductivity of certain metals at different temperatures.

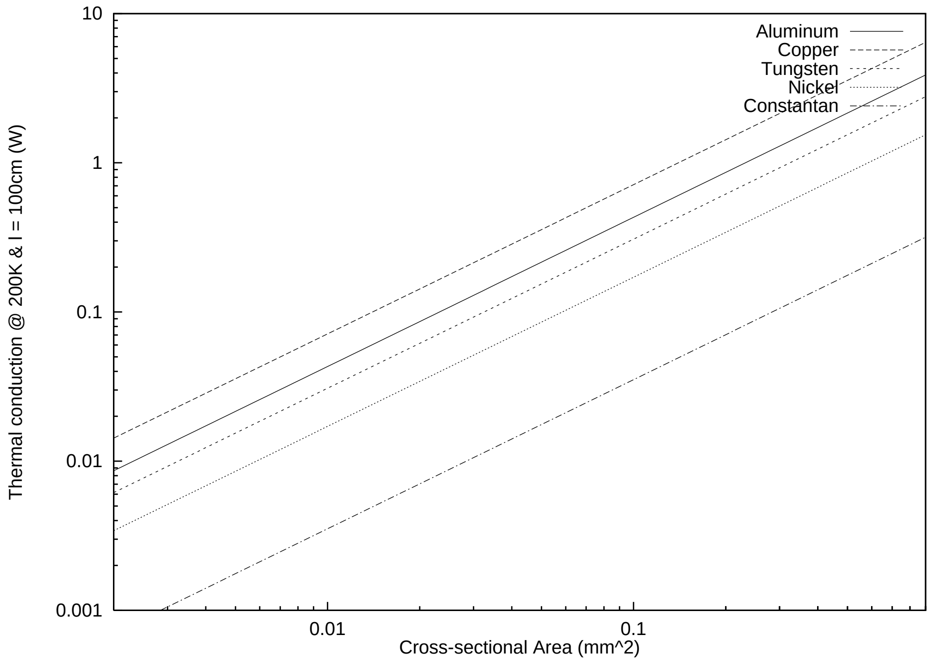

Figure 2: Thermally conducted heat for all eighty motor wires for a temperature gradient of 220K degrees, assuming a cable length of 100cm, and at an average temperature of 200K.

Resistivity of the wires

The other characteristic that has to be looked at when selecting an appropriate material for the motor wires is its resistivity which will determine for a given current the I^2R losses. For high resistivity, small cross-sectional area or long cables (or any combination of the three) the I^2R losses can dominate over the thermal heat generation.

The electrical resistivity (symbol ρ) is defined as the characteristic proportionality factor equal to the resistance of a centimeter cube of a substance to the passage of an electrical current perpendicular to the parallel faces.

$$R = \rho(l/A) \tag{3}$$

where R is the resistance of a uniform conductor, l is the length, A is its cross-sectional area, and ρ is the resistivity.

The values tabulated in table 3 give the resistivity for a range of pure metals plus the alloy constantan. It must be noted that resistivity of a metal usually increases due to impurities especially at low temperatures where the effects largely determine the approximately constant value of the so-called residual resistance to which the resistance of many metals decrease to, with decreased temperature.

Electrical Specification of NOTCam Motors

All the motors in NOTCam are Berger Lahr 5-phase stepper motors type VRDM564/50. These motors are rated at 0.95A nominal current/winding for a torque of 30Ncm.

The maximum duration for operating a motor will be set in software as a timeout and will be equal to the period to turn a wheel one full revolution plus some margin for error. This is estimated at about 10s.

Metal	Resistivity ($\mu\Omega$cm)	
	80K	273K
Aluminium	0.3	2.45
Antimony	8	39.0
Beryllium	-	2.8
Bismuth	35	107
Cadmium	-	3.43
Chromium	-	13.4
Constantan	-	48
Copper	0.2	1.56
Gold	0.5	2.04
Iridium	0.9	4.9
Iron	0.66	8.9
Lead	4.7	19.0
Magnesium	0.62	3.9
Mercury	5.8	94.1
Molybdenum	0.7	5.2
Nickel	0.55	6.14
Niobium	-	13.1
Palladium	1.73	10.0
Platinum	1.96	9.81
Silver	0.3	1.51
Tantalum	2.56	12.6
Tin	2.1	11.5(20^{o}C)
Titanium	16	50
Tungsten	0.6	4.9
Zinc	1.1	5.5

Table 3: Resistivity of metals at 80K and 273K.

The input energy into the motor wires is simply I^2R where I is the winding current (average 0.95A for maximum torque) and R is the resistance of the wire. Note the motor windings have a resistance of 1.7Ω each. See the consequence of this below.

It has been stated that, regarding the torques needed to drive the various mechanisms, except the shutter, the highest figure is 7Ncm for the focus mechanism, (and then with a very conservative calculation). From the calculations below the shutter motor needs to be driven with its full torque therefore full nominal current of 0.95A.

There is a linear relationship between current and torque. Therefore the required current for a torque of 7Ncm is 0.22A. It must be noted that Berger Lahr specify the minimum torque for the nominal current, the actual torque can be up to 30% higher. As a consequence the motors can probably be safely operated with a current of only 0.2A (this assumes a 10% under estimate of the actual torque at nominal current).

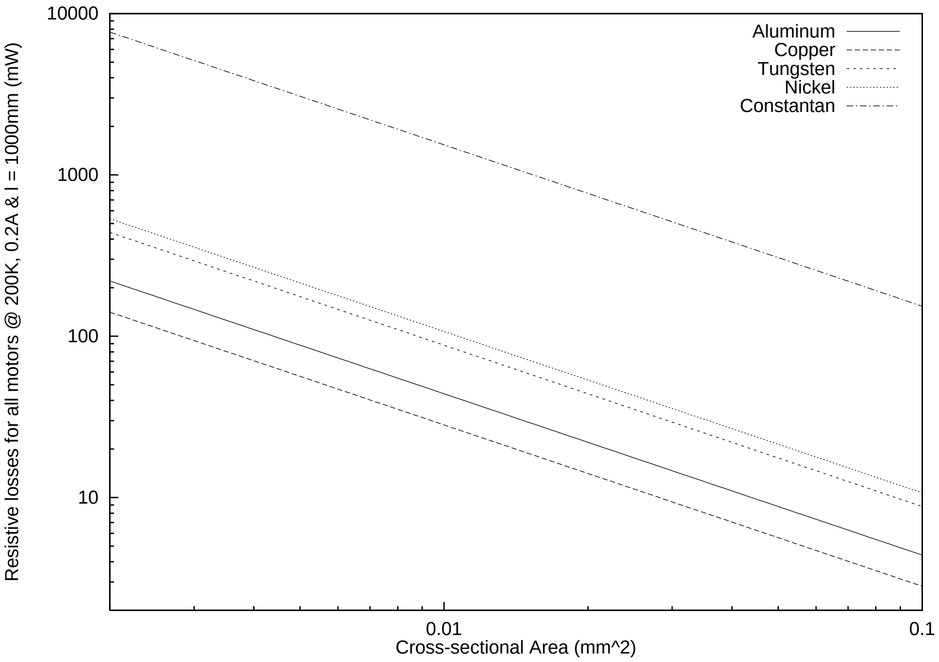

Figure 3: Resistive losses for all motors, assuming a cable length of 100cm, a nominal current of 0.2A, an average duty cycle of about 1% for each motor and at an average temperature of 200K.

The resistive load can be determined from;

$$P = \rho_{200K}(l/A)I^2 D N_w N_m \tag{4}$$

where D is the duty cycle of each motor (1%), N_w is the number wires per motor $= 10$, l is the length $=$ 100cm, I the nominal current $=$ 0.2A and N_m is the number of motors $= 8$.

From equation 4 Figure 3 shows what resistive losses you will get for a nominal current of 0.2A, for all the motors with an average duty cycle of 10% (i.e. just over 1% each) and at an average temperature of 200K against cross-sectional area of the wires for different wire materials. From figure 3 it can been seen that where constantan was the ideal material to minimise the thermal losses it is by far the worst for resistive losses. This is due to its very high resistivity of 48$\mu\Omega$cm compared with just over 2$\mu\Omega$cm for copper.

Calculation of shutter motor torque requirements

The shutter has an inertia of 0.25kgcm^2 and the motor has a rotor inertia of 0.08kgcm^2 making a total of 0.33kgcm^2.

The quoted maximum torque, T, is 30Ncm, given the inertia, J, of 0.33kgcm^2 then the shutter can be accelerated at $a = T/J = 30/0.33 = 90 rads/sec^2$. The time it takes for the shutter to rotate a quarter turn can be calculated from $at^2/2 = \omega$ where t is the time and ω is the angle of rotation in radians. Therefore a turn of $2\pi/4$ gives $\sqrt{(2\pi/4)2/a} = 187ms$. This means that the shutter motor needs to operate at its maximum torque to give a satisfactory operating speed. All the other motors can be operated at the lower torque and hence a low drive current.

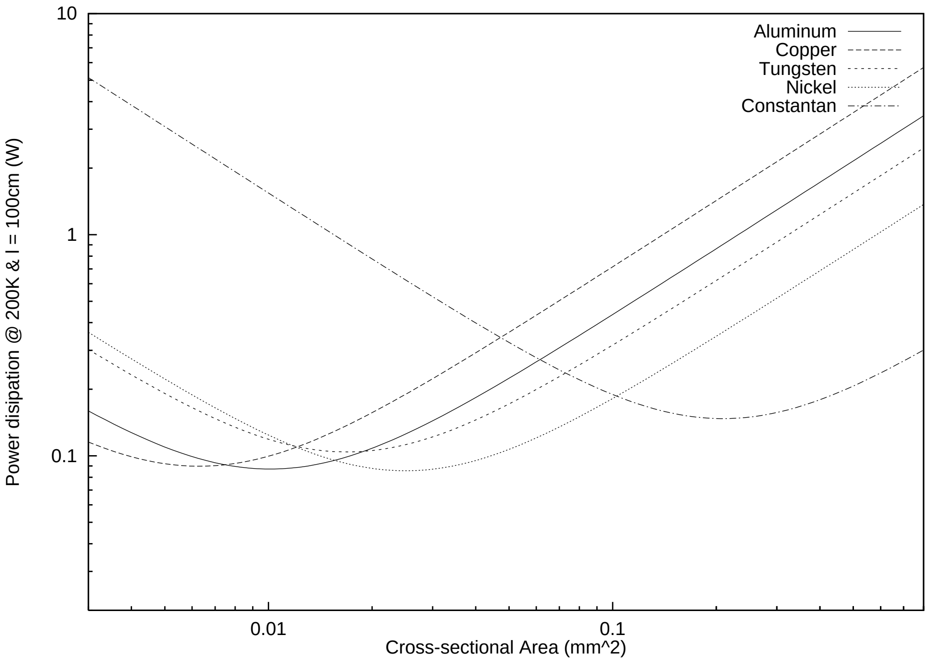

Figure 4: Sum of both thermal and resistive heat losses, assuming a cable length of 100cm, a nominal current of 0.2A, a duty cycle of 1% for each motor and at an average temperature of 200K.

Heat Budget

From above it has been shown that the cross-sectional area and selection of wire materials both have a strong influence on the choice of cable that ideally should be used to minimise either the thermal conduction or the resistive losses. Unfortunately these two parameters are not complementary and a compromise has to be found where you get the minimum heat load for a particular cross-sectional area for the best material.

From equations 2 and 4 the total heat load can be determined. Total heat load is therefore given by:

$$W = \frac{K_{200K}(t_1 - t_2)AN_wN_m}{l} + \rho_{200K}(l/A)I^2DN_wN_m \tag{5}$$

For A in centimetres, equation 5 reduces to;

$$W = 176K_{200K}A + \frac{3.2\rho_{200K}}{A} \tag{6}$$

In figure 4 is plotted the total sum of the thermal and resistive heat load against the cross-sectional area. From figure 4 it can be seen by simply changing the cross-sectional area for any of the proposed wire materials a substantially difference is made to the source and amount of heat load contributed by the wires. For small cross-sectional areas the resistive losses dominate and as you increase the area these fall and the thermal contribution takes over. The minimums of these curves can be calculated by differentiating equation 5 for the total power losses. This gives;

$$A = Il\sqrt{\frac{\rho_{200K}D}{K_{200K}(t_1 - t_2)}} \tag{7}$$

Material	K_{200K} $Wcm^{-1}C^{-1}$	ρ_{200K} $\mu\Omega$cm	csa mm^2	P_{min} mW
Aluminium	2.45	1.375	0.01	87
Copper	4.05	0.88	0.0063	90
Tungsten	1.75	2.75	0.017	104
Nickel	0.97	3.35	0.025	85
Constantan	0.2	48	0.21	147

Table 4: The absolute minimum heat load expected with the corresponding cross-sectional area for the NOTCam eight internal motors cables for five selected wire materials.

l_{cu} mm	A_{cu} mm^2	Dia. mm
500	0.0031	0.06
1000	0.0063	0.09
3000	0.019	0.15
5000	0.031	0.2
7000	0.044	0.24
9000	0.056	0.27
10000	0.063	0.28

Table 5: This table shows a range of cross-sectional areas and corresponding lengths that give the same absolute minimum heat load expected for copper (Cu) of approximately 90mW.

which reduces, for A in millimetres, to;

$$A = 13.5\sqrt{\frac{\rho}{K_{200K}}} \tag{8}$$

Similarly by substituting the cross-sectional area back into equation 5 the minimum power can be determined. Table 4 shows the absolute minimum heat load you can expect for 1m long wires for the five selected materials.

From table 4 and the curves in figure 4 for the correct material and cross-sectional area an average power loss between 85mW and 100mW can be obtained. From figure 4 it can be seen that it does not make a great deal of difference which out of the pure metals one uses. They all give around the same minimum heat load for approximately the same cross-sectional area.

Further if you substitute the cross-sectional area for minimum heat load in to equation 7 it is possible to obtain the corresponding lengths of wire for any combination. Table 5 give an example of the range of wire lengths for copper for minimum power load. If you take any pair of numbers and put them in equation 5 you will find the minimum power load is always the same.

If we take the case for copper wires in NOTCam with eighty motor wires and a nominal current of 0.2A with a duty cycle of 1% for each motor then as stated in table 4 the minimum heat load is 90mW.

For the values quoted in table 4 the resistance of the copper wire is 1.4Ω, in all cases, which with a current of 0.2A gives a volt drop of less than 0.3V.

A graph of the power dissipation from copper wire for lengths from 0.5m to 10m and with the corresponding cross-sectional area is shown in figure 5. This graph has the typical shape of all the discussed wire materials though of course the axes will change.

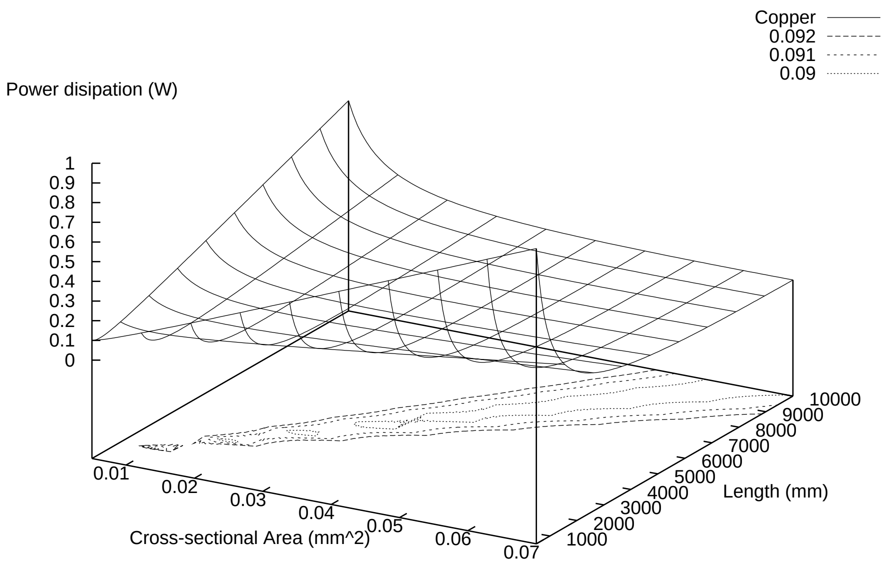

Figure 5: Sum of both thermal and resistive heat load for copper, for a range of wire lengths and cross-sectional areas, a nominal current of 0.2A, a duty cycle of 1% for each motor and at a average temperature of 200K. The contours for minimum power load are also displayed.

As mentioned earlier the shutter motor needs to be operated at it full power. This means a current of 0.95A. Copper stranded ribbon cable with wires of cross-sectional area 0.08mm^2 can handle this. Using equation 5 the heat load for this configuration can be determined. The thermal load comes to 72mW and as before assuming a duty cycle of 1% then the resistive contribution is 11mW making a total heat load from the shutter motor wires of 83mW.

One item to note is that the materials have been assumed to be pure metals but probably wires used will not be, so higher resistive looses will be expected.

It must not be forgotten that in addition to the wire losses each winding (phase) of the stepper motors have a resistance of 1.7Ω. This resistance gives a I^2R loss for each motor of 7.67W at a current of 0.95A or 0.34W at 0.2A.

From the figures for the actual motor windings power dissipation it can been seen that the shutter motor is going to generate a lot of heat, especially since it will probably have a larger duty cycle than any of the wheel or focus motors.

Wire temperature

As a check it is necessary to calculate what temperature a thin wire would reach in operation.

If we assume for the filter and focus motors we use copper wire of cross-sectional area 0.05mm^2 and length 100cm then what temperature will it be raised to if it passes a current of 0.2A for a duration of 10s, the expected maximum period to rotate one of the wheels.

The change in temperature is given by;

$$\Delta T = \frac{Q}{mc_{cu}} \tag{9}$$

where ΔT is the increase in temperature, Q is change in heat content (joules), m is the mass and c_{cu} the specific heat capacity of copper in (Jg^{-}1K^{-1}).

The density of copper is $\rho_{cu} = 8.94$ gcm^{-3}, therefore the mass of the 1m of wire is 0.45g.

At 0.2A the I^{2}R power loss in the wire is 7mW.

The specific heat capacity of copper is 0.385 Jg^{-1}K^{-1}. From equation 9 ΔT = 7x10^{-3} / (0.45 x 0.385) = 0.04Ks^{-1} or 0.4^{o}C in 10s.

Repeating the above calculation for the shutter motor wires. For the shutter, copper wires of cross-sectional area 0.08mm^2 can be used for the larger current of 0.95A, then ΔT = 0.1 / (0.715 x 0.385) = 0.363Ks^{-1}. The shutter only operates for 0.2s to open or close resulting in a ΔT of 0.073^{o}C.

Conclusions

It can be concluded as long as you select a wire of the right cross-sectional area, length and material then the wire heat load can easily be kept to a few hundred milli-watts against the 2Watts originally expected.

From figure 4 it can be seen that copper wire can for a reasonable cross-sectional area provide a satisfactory result. If we assume that we use copper stranded ribbon cable, with wires of cross-sectional area 0.08mm^2 and a length of 1m for all the motors and for a duty cycle of 1%, then using equation 5 we get a total minimum heat load of around 600mW. If instead 0.05mm^2 copper insulated wire, as used in transformers etc., is chosen for all the motors, except the shutter motor that needs the thicker 0.08mm^2 cable, then a minimum heat load of only 450mW can be obtained, 83mW from the shutter motor wires plus 360mW from the other seven motors. Also the increase in temperature even with these thin wires is negligible. For a copper wire of cross-sectional area 0.05mm^2 and a current of 0.2A, a change in ΔT of only 0.4^{o}C occurs.

If necessary better results can be obtained using any of the other wire materials. For example constantan with a relatively large cross-sectional area of 0.5mm^2, (to minimise resistive losses), you only have, from figure 4, potentially about 200mW heat load, almost entirely thermal, but with a 1volt drop along the length at 1A.

Bibliography

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